



## MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- This Question Paper consists of three sections A, B and C.
- Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.
- Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.
- Section B: Internal choice has been provided in one question of two marks and one question of four marks.
- Section C: Internal choice has been provided in one question of two marks and one question of four marks.
- All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
- The intended marks for questions or parts of questions are given in brackets [ ].
- Mathematical tables and graph papers are provided.

### SECTION A - 65 MARKS

#### Question - 1

(i) If  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ , then  $A^2$  is equal to

- (a)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- (b)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (c)  $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$
- (d)  $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

(ii) If  $\int \tan^4 x \, dx = a \tan^3 x + b \tan x + cx + C$ , then find the values of a, b, and c.

- (a)  $a = \frac{1}{2}, b = -1, c = 1$
- (b)  $a = \frac{1}{3}, b = -1, c = 1$

(c)  $a = 1, b = -1, c = 2$

(d)  $a = \frac{1}{4}, b = -1, c = 1$

(iii) The value of  $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$  is.

- (a) 1
- (b) 2
- (c) -3
- (d) -1

(iv)  $\frac{dy}{dx} = \frac{y(xy-x^2)}{x(x^n+y^n)}$

- (a)  $n = 1$
- (b)  $n = 2$
- (c)  $n = -3$
- (d)  $n = -1$

(v) A and B are two independent events such that  $0 < P(A) < 1$ ,  $0 < P(B) < 1$ , then which of the following is not correct?

- (a) A and B are mutually exclusive
- (b) A and  $B'$  are independent
- (c)  $A'$  and B are independent
- (d)  $A'$  and  $B'$  are independent

(vi) If the following function  $f(x)$  is continuous at  $x = 0$   $\left| \begin{array}{ccc} a & b & c \\ x+a & y+2a & z+3a \\ 1 & 2 & 3 \end{array} \right| =$

$\left| \begin{array}{ccc} a & b & c \\ x & y & z \\ 1 & 2 & 3 \end{array} \right| + |B|$ , then the value of  $|B|$  is equal to

- (a)  $a + 2b + 3c$
- (b)  $a - b + 2c$
- (c) 0
- (d)  $3a$

(vii) The function  $f(x) = e^{|x|}$  is

- (a) Continuous everywhere but not differentiable at  $x = 0$
- (b) Continuous and differentiable everywhere
- (c) Not continuous at  $x = 0$
- (d) None of the above

(viii) The function  $f: R \rightarrow R$  given by  $f(x) = -|x - 1|$  is

- (a) Continuous as well as differentiable at  $x = 1$
- (b) Not continuous but differentiable at  $x = 1$
- (c) Continuous but not differentiable at  $x = 1$
- (d) Neither continuous nor differentiable at  $x = 1$

(ix) Consider  $f: R_+ \rightarrow (4, \infty)$  given by  $f(x) = x^2 + 4$ , where  $R_+$  denotes the set of all positive real numbers.

- (a) one-one but not onto.
- (b) onto but not one-one.
- (c) Both one-one and onto.
- (d) Neither one-one nor onto.

(x) Assertion (A): The sum of diagonal elements of a skew-symmetric matrix is zero.

Reason (R): All the diagonal elements of a skew-symmetric matrix are zero.

- (a) Both A and R are correct and R is the correct explanation of A.  
 (b) Both A and R are correct but R is not the correct explanation of A.  
 (c) A is correct but R is not correct.  
 (d) A is not correct but R is correct.

- (xi) Let R be an equivalence relation in the set  $A = \{0, 1, 2, 3, 4, 5\}$  given by  $R = \{(a, b) : 2 \text{ divides } (a, b)\}$ . Write the equivalence class  $[0]$ .
- (xii) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$  is matrix which satisfies  $AA' = 9I_3$ , then find the values of a and b.
- (xiii) Let  $A = \{1, 2, 3, 4\}$ ,  $B = \{5, 6, 7, 8, 9\}$  and  $F: A \rightarrow B$  be defined by  $f = \{(1, 5), (2, 6), (3, 7), (4, 8)\}$ . Then, f is  
 (a) One-one but not onto  
 (b) Onto but not one-one  
 (c) Neither one-one nor onto  
 (d) One-one and onto
- (xiv) Three persons A, B and C attempt a Mathematics problem independently. The odds in favour of A and C solving the problem are 3: 2 and 4: 1 respectively and the odds against B solving the problem are 2: 1. The probability that problem will be solved is
- (xv) If  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cup B) = \frac{5}{6}$  then events A and B are.....

## Question 2

- (i) Find the derivative of  $f(e^{\tan x})$  w.r.t. x at  $x = 0$ . It is given that  $f'(1) = 5$ .  
 OR  
 (ii) Prove that the function  $f(x) = x - x + 1$  is neither increasing nor decreasing on  $(-1, 1)$ .

## Question 3

Evaluate :  $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$ , find a.

## Question 4

At what points on the circle  $x^2 + y^2 - 2x - 4y + 1 = 0$ , the tangents are parallel to x-axis?

## Question 5

- (i) Evaluate :  $\int \frac{dx}{x \cos^2(1+\log x)}$

OR

- (ii) Evaluate :  $\int \frac{\cos^{-1} x}{x^2} dx$

## Question 6

Find the domain of the function f given by  $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$

## Question 7

Solve for  $x$ :  $\cos^{-1}(\sin(\cos^{-1} x)) = \frac{\pi}{3}$

### Question 8

Evaluate:  $\int \frac{6x+5}{\sqrt{6+x-2x^2}} dx$ .

### Question 9

(i) The function  $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 - x & \text{if } 1 \leq x \leq 2 \\ -2 + 3x - x^2 & \text{if } x > 2 \end{cases}$  is .....

OR

(ii) If  $e^y(x+1) = 1$ , show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ .

### Question 10

Each friend rolls a fair die. If the sum of their three rolls is even, they toss again; if odd, they pay equally.

- (a) Probability sum is odd in one round?
- (b) Probability they need more than 2 rounds to decide?

OR

Three Age Groups, Survey on usage of a new feature:

- Teens: 30% of users, adoption rate 50%
- Adults: 50% of users, adoption rate 20%
- Seniors: 20% of users, adoption rate 10%

A random user is found to use the new feature.

- (a) Represent with probability notation.
- (b) Find probability the user is a Teen.
- (c) Which age group is most likely given they use the feature?

### Question 11

A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

### Question 12

- (i) Find the particular solution of the differential equation  $\frac{dy}{dx} + y \tan x = 3x^2 + x^3 \tan x$ , given that  $y = 0$  when  $x = \frac{\pi}{3}$ .

OR

- (ii) Evaluate:  $\int \frac{x^2 - 5x - 1}{x + x^2 + 1} dx$ .

### Question 13

- (i) A given quantity of metal is to be cast into a half circular cylinder (i.e. with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is  $\pi : (\pi + 2)$ .

OR

- (ii) Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere radius  $r$  is  $\frac{4r}{3}$ . Also show that the maximum volume of the cone is  $\frac{8}{27}$  of the volume of the sphere.

### Question 14

The probability distribution of a random variable  $X$  is given as below:

$$P(X = x) = \begin{cases} k(x + 1), & \text{for } x = 1, 2, 3, 4 \\ 2kx, & \text{for } x = 5, 6, 7 \\ 0, & \text{for } x = \text{otherwise} \end{cases} \quad \text{where } k \text{ is a constant. Calculate : (i) the value of } k$$

- (ii)  $E(X)$ .

### SECTION B 15 MARKS

### Question -15

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) The vector in the direction of the vector  $\hat{i} - 2\hat{j} + 2\hat{k}$  that has magnitude 9 is

- (a)  $\hat{i} - 2\hat{j} + 2\hat{k}$   
 (b)  $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$   
 (c)  $3(\hat{i} - 2\hat{j} + 2\hat{k})$   
 (d)  $9(\hat{i} - 2\hat{j} + 2\hat{k})$

- (ii) The two planes  $x - 2y + 4z = 10$  and  $18x + 17y + kz = 50$  are perpendicular, if  $k$  is

- (a) -4  
 (b) 4  
 (c) 2  
 (d) -2

- (iii) The two vector  $\hat{j} + \hat{k}$  and  $3\hat{i} - \hat{j} + 4\hat{k}$  represent the two sides  $AB$  and  $AC$ , respectively of  $\triangle ABC$ . Find the length of the median through  $A$ .

- (iv) If  $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$  and  $|\vec{a}| = 5$ , then write the value of  $|\vec{b}|$ .

- (v) Find the sum of intercepts cut off by the plane  $2x + y - z = 5$  on the coordinate axes.

### Question -16



Find the image of the point  $(3, -2, 1)$  in the plane  $3x - y + 4z = 2$

OR

Write the distance between the parallel planes  $2x - y + 3z = 4$  and  $2x - y + 3z = 18$ .

#### Question -17

Find the value of  $p$ , so that the lines  $l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$  and  $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are perpendicular to each other.

OR

Find the equation of the plane passing through the line of intersection of the plane  $x + y + 3z - 5 = 0$  and  $3x - 2y + z + 1 = 0$  and cutting off equal intercepts on the X axis and Z axis.

#### Question -18

Find the area of the figure, bounded by the graph functions  $y = x^2$  and  $y = 2x - x^2$ .

### SECTION C 15 MARKS

#### Question -19

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) If the demand function for a monopolist is given by  $x = 100 - 4p$ , the average revenue function is  
 (a)  $25 - \frac{x}{4}$  (b)  $25 - \frac{x^2}{4}$  (c)  $25 - \frac{x}{2}$  (d) None of these
- (ii) If  $\sum x = 30$ ,  $\sum y = 15$ ,  $\sum xy = 94$  and  $\sum x^2 = 190$ , then  $b_{yx}$  is  
 (a) 0, 6 (b) 5 (c) 1 (d) 0
- (iii) The fixed cost of a new product is Rs 18000 and the variable cost per unit is Rs. 550. If the demand function is  $p(x) = 400 - 150x$ , find the break even values.
- (iv) The total revenue received from the sale of  $x$  units of a product is given by  $R(x) = 24x + 4x^2 + 12$ , find average revenue.
- (v) Two lines are  $2x - 4y = 0$  and  $x - 3y + 5 = 0$ . Find the correlation coefficient between  $x$  and  $y$ .

#### Question -20

- (a) The selling price of a commodity is fixed at Rs. 60 and its cost function is  $C(x) = 35x + 250$   
 (I) Determine the profit function (II) Find the break even points.

OR

- (b) The revenue function is given by  $R(x) = 100x - x^2 - x^3$  Find

- (I) The demand function. (II) Marginal revenue function.

#### Question -21

The following results were obtained with respect to two variables  $x$  and  $y$ :  $\sum x = 15$ ,  $\sum y = 25$ ,  $\sum xy = 83$  and  $\sum x^2 = 55$ ,  $\sum y^2 = 135$  and  $n = 5$  (I) Find the regression coefficient  $b_{xy}$  (II) Find the regression equation of  $x$  on  $y$ .

### Question -22

A manufacturer wishes to produce two commodities A and B. The number of units of material, labour and equipment needed to produce one unit of each commodity is shown in the table given below. Also shown is the available number of units of each item, material, labour and equipment.

| Items     | Commodity A | Commodity B | Availability |
|-----------|-------------|-------------|--------------|
| Material  | 1           | 2           | 8            |
| Labour    | 3           | 2           | 12           |
| Equipment | 1           | 1           | 10           |

Find the maximum profit if each unit of commodity A earns a profit of Rs. 2 and each unit of B earns a profit of Rs 3.

OR

A manufacturer makes two products A and B. Product A sells at Rs. 200 each and takes 1/2 hour to make product B sells Rs.300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consist of 40 hours of production and the weekly turnover must not be less than Rs. 10,000. If the profit on each of product A is Rs. 20 and on product B is Rs. 30, then how many of each should be produced so that the profit is maximum? Also find the maximum profit.